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# OPTIMAL DECISION-MAKING IN MANUFACTURING INDUSTRY UNDER BUSINESS RISK

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***Abstract:*** In the modern global economy, emerging economies have become the main source of manufactured goods to supply the advanced economies. The profits of a manufacturing firm normally increase in the amount it can produce and sell. However, in an environment of uncertainty, it is not always optimal for a manufacturing firm in emerging economies to accept big production orders even though this might mean higher revenues. This is because a corresponding higher operating leverage will necessarily increase business risk and the resulting discount rate, which may result in a lower firm value. In this study, we provide a structural framework targeting this problem where demand is stochastic, discount rate is endogenously determined based on business risk and operating leverage. We use lithium battery, an essential component in electric and hybrid cars, to demonstrate how the framework is developed and implemented. Our model helps decide the optimal production capacity as the result of the cost structure, risk aversion, and value maximization. We show how the valuation process is carried out in a theoretically consistent manner. Our framework may be extended for different sectors in emerging economies when production decisions are to be made under uncertainty.

***JEL classification:*** G10, G12, G30

***Keywords:*** optimal capacity, business risk, operating leverage, lithium battery

## I. INTRODUCTION

The motivation of this study comes from a puzzle that I have held through personal experience and observation. As of April 2022, there are more than one million registered firms and about 60,000 factories in the city of Dongguan, Guangdong Province, China. Most of the electronic devices are produced here including almost

100% iPhones and iPads. More than 90% of the laptops sold world-wide are produced in Chongqing City. More than 50% locks, children's clothes, 80% lighters, and almost 100% water crystal ash trays sold world-wide are produced by many small shops or companies in a few towns in Zhejiang Province. Typically, we observe many small shops or firms concentrated in one city specializing in making one type of product. These firms normally compete with each other, and they provide most of the items that Wal-Mart, Target, JC Penny, etc. sell globally. It is sometimes called the "Dongguan-Wal-Mary Effect." It is interesting to note that after so many years, there are still not big firms emerging from those myriad small manufacturers. A few years ago, the owner of a small Dongguan firm told me that many of them would refuse to take big orders. She admitted that she does know exactly why beyond gut feeling. Ever since then it has been an interesting puzzle for me as well. My puzzle may be rephrased as follows. Why would not these small firms making similar products merge into bigger firms thereby they would have a much bigger bargaining power when negotiating with the powerful downstream demand side in the supply chain such as Wal-Mart? In other words, what rational concerns prevent them from growing? My study reveals that the reason lies in the type of business risk and the way it is managed in the production process.

In the modern global economy, advanced economies are the demand side located downstream in the international supply chain structure and emerging economies tend to be upstream and serve as the supply side by manufacturing goods consumed by the advanced economies. Normally, the profits of the manufacturers increase in the size of the demand. It always appears to be a good thing for a business to receive orders from its customers, and perhaps the larger the better as our usual intuition goes. However, it may not be a wise decision to accept every order even though it will boost the revenue. This is because a bigger scale of operation and production may entail an expansion of facilities which increases fixed cost. If the order size fluctuates, higher fixed costs will become a heavier burden especially when order size drops. Therefore, when to happily accept an order and when to politely "say no" to your customer is a matter of corporate decision of the highest concern in risk management. The issue we have described boils down to the question of optimal production capacity when facing uncertain demand and changing price. The main challenges come from two sources: volatile revenue and the appropriate discount rate. Out of these two challenges, determining the appropriate discount rate is even a bigger challenge. Within the context of the CAPM, it becomes the determination of the changing risk measure, i.e., beta, which is endogenously related to operating leverage, or fixed and variable costs, and demand, and exogenously related to external stochastic driving forces. In this study, we use lithium battery industry as an example where we attribute the stochastic driving force to the fluctuating oil price.

### 1.1. Lithium Battery Industry

Lithium batteries are widely used in electric and hybrid cars. The global lithium-ion battery market size was $37 billion in 2020 and expected to grow to $193 billion by 2028. According to the current market and technology, batteries are the most critical component in terms of both economic value (or cost) and engineering hurdle for promoting electric and hybrid automobiles. Since the size of the global automobile market is enormous, lithium battery's market would heavily depend on the growth of energy-efficient cars whose popularity is closely correlated with oil price. Empirical evidence supports the intuition that higher oil price and greater price volatility tend to boost the sales of energy-efficient automobiles. Therefore, the market for lithium batteries is approximately a function of oil price which is volatile. To produce lithium batteries, in addition to this market uncertainty, one must also consider how variable and fixed costs and production scale interact with market uncertainty. These factors pose a difficult challenge to the decision-makers in deciding whether to produce lithium batteries or not. In this study, we propose a theoretical framework to decide the critical lithium price at which investing in lithium battery production is an optimal decision. In particular, we

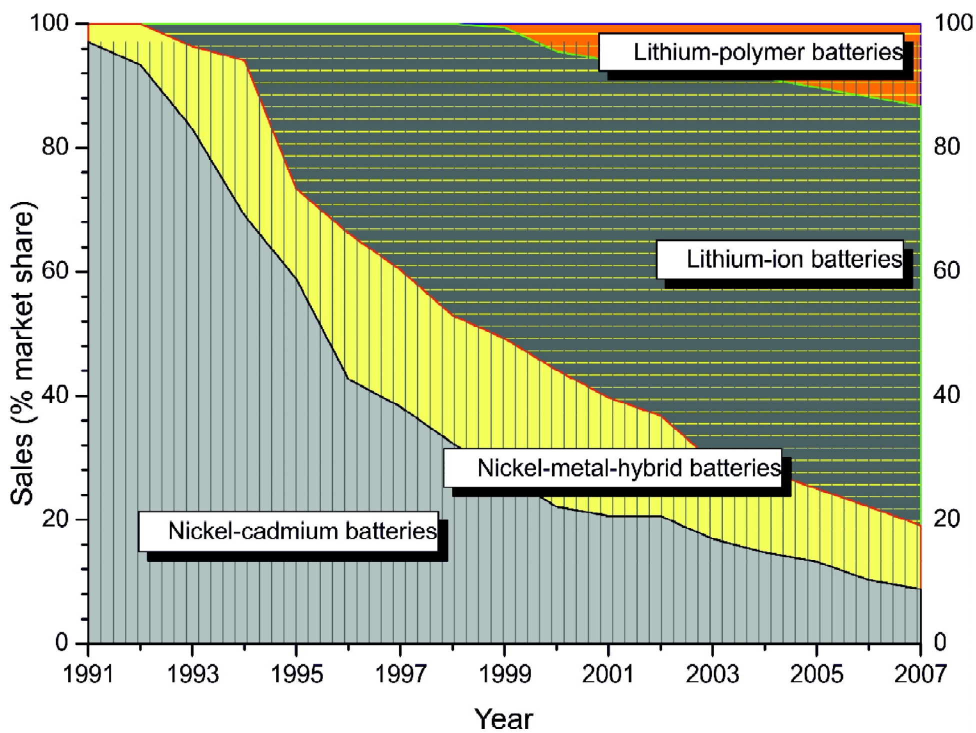


**Figure 1: Global sales of rechargeable batteries from 1991 to 2007 as a percentage of total global sales of rechargeable batteries. Source: Goonan (2011). Original data are from Wilburn (2007) and Takashita (2008)**

investigate the relationship between this critical lithium price and the business expansion plan. Our framework is flexible enough to incorporate more relevant factors and general enough to adapt to other situations of enterprise development under uncertainty after modification.

## II. THE MODEL: OPTIMIZING THE PRODUCTION CAPACITY

Since a new technology is constantly facing competition and eventually become outdated in the sense that the annual profit falls to barely cover the variable cost or even becomes negative with and without further considering fixed cost, we set a realistic time horizon $H$ years for the lithium battery project. Our question is now – what is the optimal production capacity our firm should have when it is facing stochastic demand and falling battery price? This is formulated in the following equation

$$Q^*_{Capacity} = \underset{Q_{Capacity}}{Arg\ Max}\left[V_T\right] = \underset{Q_{Capacity}}{Arg\ Max}\left[\sum_{t=1}^{T} PV\left(CF_t\right)\right], \quad s.t. \quad T \leq H \tag{1a}$$

Or in a statistical formulation,

$$E\left[Q^*_{Capacity}\right] = \underset{Q_{Capacity}}{Arg\ Max}\left[E\left[V_T\right]\right] \tag{1b}$$

Where $Q^*_{Capacity}$ is the optimal production capacity, $T$ is the expected life of the project, $E\left[V_T\right]$ is the *total* expected present value of the entire lithium project. For the asset cash flow $CF_t$ at $t$, the present value is

$$PV\left(CF_t\right) = \frac{I_t Q_t p_t - I_t Q_t VC_t - FC_t}{\prod_{i=1}^{t}\left(1 + r_i\right)} = \frac{I_t Q_t p_t - I_t Q_t VC_t - a_f Q_{Capacity}}{\left(1 + \Re_t\right)^t} \tag{2}$$

Where $p_t$ is the battery price (\$/kWh), $VC_t$ is the variable cost, $FC_t$ is the fixed cost, $Q_t$ is the stochastic demand, and $\Re_t$ is the appropriate discount rate, i.e., $t$-year spot rate. We note that all of these variables are functions of time $t$, and some are stochastic driven by stochastic oil price $p_{oil\_t}$.

The dynamic decision-making in (2), i.e., whether to accept a production order from our downstream customers, is reflected in the index function given as follows

$$I_t = I_t\left(\frac{Q_{Capacity} - Q_t}{Abs\left[Q_{Capacity} - Q_t\right]} > 0\right) = \frac{Max\left[Q_{Capacity} - Q_t, \quad 0\right]}{Q_{Capacity} - Q_t} \tag{3}$$

## III. SETTINGS AND VARIABLES IN THE MODEL

In this section, we explain the details of how all the variables are determined after we specify the stochastic oil price process.

### 3.1. Stochastic Factor: Oil Price Time Series $\{p_{oil\_0},\ p_{oil\_1},\ \ldots,\ p_{oil\_t},\ \ldots,\ p_{oil\_T}\}$

Modeling oil price can be a challenge and as well as a judgment call depending on our time horizon, purpose, accuracy requirement, etc. For example, Pindyke (1999) showed that the Geometric Brownian Motion (GBM) assumption works well when the implied volatility is relatively constant. Dixit and Pindyke (1994) considered mean-reverting process under the GBM assumption. Postali and Picchetti (2006) demonstrated that GBM can perform well as a proxy for the movement of oil prices. In our model, oil price is the dominant stochastic factor that directly affects other secondary dependent factors, such as the market demand, production costs (variable and fixed), inventory cost, etc. Oil price $p_{oil\_t}$ is modeled to follow a geometric Brownian motion:

$$dp_{oil\_t} = \mu_{oil}\, p_{oil\_t} dt + \sigma_{oil}\, p_{oil\_t} dW_t \tag{4}$$

Or in discrete form,

$$\Delta p_{oil\_t} = p_{oil\_(t+1)} - p_{oil\_t} = \mu_{oil}\, p_{oil\_t} \Delta t + \sigma_{oil}\, p_{oil\_t} \Delta W_t \tag{5}$$

where $\mu_{oil}$ is the long-term growth rate of oil price; $\sigma_{oil}$ is the volatility of oil price; $\Delta W_{t-1}$ is the standard Brownian stochastic movement with mean 0 and volatility 1. Here we assume that $\mu_{oil}$ is constant unless we have more reliable information to indicate otherwise.

### 3.2. Demand $Q_t$

In the late 20$^{th}$ century, lithium became important as an anode material in lithium batteries because of its desirable chemical and physical properties, such as being the lightest metal. Its high electrochemical potential makes it a valuable component of high energy-density rechargeable batteries. Lithium batteries are also widely used in powering laptop computers, cordless heavy-duty power tools, and hand-held electronic devices, etc. An even greater lithium market is expected as electric and hybrid vehicles are growing fast and alternative energy production receives more emphasis as oil becomes more costly in terms of price as well as volatility.

Electric cars have three main types - all electric (EV), hybrid (HEV), or plug-in hybrid (PHEV) vehicles. The U.S. Government planned to provide $11 billion to help car and battery makers to reduce the dependence on foreign oil (See, e.g., Smith and Craze, 2009). The main reasons are reducing the dependence on imports of oil and the carbon emission from using internal-combustion engines in current automobile industry products. According to Gains and Nelson (2010), it is expected to have about 1.6 billion electric-drive vehicles built by 2050 with pure EVs accounting for over 20% of global sales.

Therefore, given a potential large lithium market, it is reasonable to assume there are many producers of lithium batteries and the "firm" in our study ("our firm" hereafter) is only one of many and a price taker. We may further assume that our firm has the same technology as other players in the industry, hence the market for our firm's products is about a fixed proportion of the total lithium battery market. Since the size of the lithium battery market may be largely determined by the popularity of electric and hybrid cars which is directly affected by oil price, we assume that the demand of our firm's products is a function of oil price. Figure 2 shows the sweet crude oil price and sales of electric cars from 2000 to 2009.

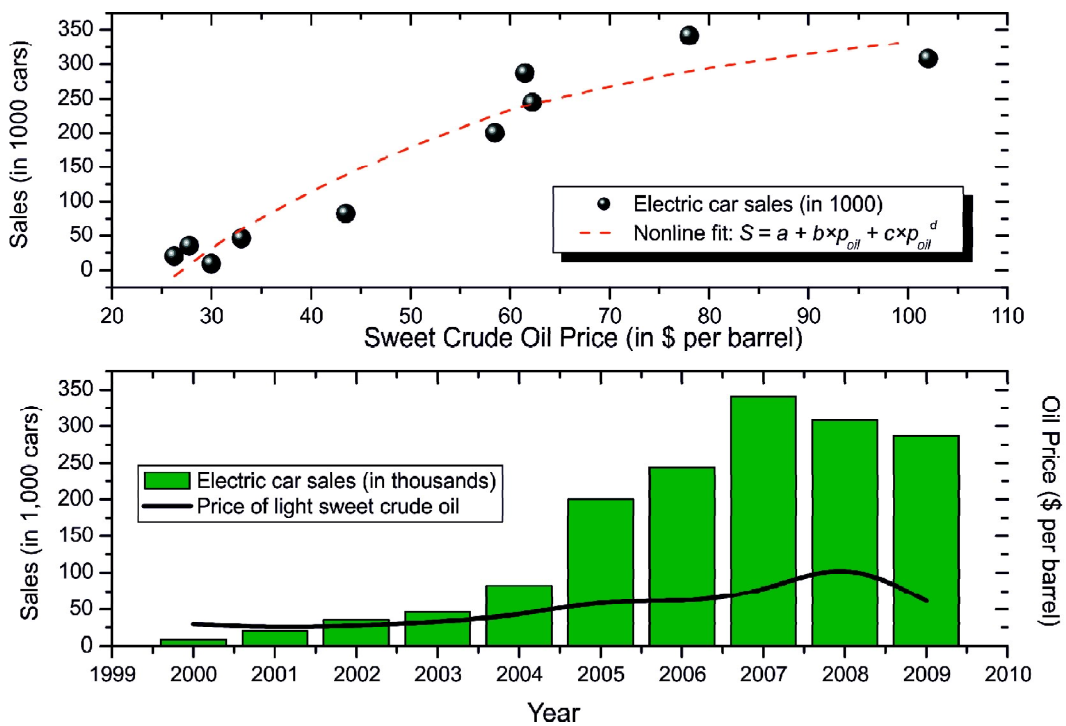


**Figure 2: Sales of hybrid automobiles in the U.S. is closely correlated with the price of light sweet crude oil from 2000 to 2009 (Goonan, 2011). Original data are from Hsiao (2008), Hybrid Cars (2010), and U.S. Department of Energy (2010)**

The corresponding relation between the two is very clear. As the oil price goes up, sales of electric-drive vehicles tend to go up too. Ideally, the relationship between oil price and our firm's demand should be calibrated against empirical data. As an illustration of our framework, we focus on its implementation rather than identifying this relationship accurately which might be a separate effort. Here, we note that the price growth rate is about half of the growth of sales of hybrid cars. Thus, we assume the relationship between oil price and the demand of the lithium batteries made by our firm takes the following form,

$$Q_t = f\left(p_{oil\_t}\right) \approx a_e + b_e p_{oil\_t} + c_e \sqrt{p_{oil\_t}} \tag{6}$$

### 3.3. Fixed and Variable Cost

Inventory cost may be a considerable part of the total costs. We also notice that more and more firms are implementing the so-called "on-demand" inventory policy, which is that the firm only produces the products according to the order it receives. Therefore, for simplicity, we assume away inventory and its costs in the current study. For future research, it is interesting to see how inventory can be incorporated in our framework and decide the optimal inventory and revisit the current study with inventory considered.

For fixed cost, *FC*, we assume it is an increasing function of our firm's total production capability. For simplicity, in this study, we assume a simple strict linear function in our firm's capacity $Q_{Capacity}$

$$FC_t = a_f \times Q_{Capacity} \tag{7}$$

where $a_f > 0$ is the fixed cost per unit. We note that $FC_t$ is not a function of the actual production quantity $Q_t$. We also note that fixed cost may also be somewhat sensitive to oil price at least in the long run, because an increase in oil price tends to drive up overall costs in various sectors. For example, if the production facility involves capital leases in equipment or real estate, the lease payments are expected to increase as a result at the time when the lease contracts are renewed. The impact of oil price on fixed cost can be introduced in (6) based on actual assessment in specific cases. Nevertheless, for simplicity, we assume our firm's fixed cost is independent of oil price fluctuations.

Variable cost may also be assumed to be a constant on a per-unit basis. However, we note that it tends to be more sensitive than fixed cost to the underlying stochastic driving force. This is because our suppliers are likely to transfer a portion of the oil price increase to our firm. For simplicity, we assume such transfers are without lags in time. Since our suppliers normally share only part of the increases in energy cost, we specify the following form of variable cost,

$$VC_t = g\left(p_{oil\_t}\right) \approx a_V + b_V \ln p_{oil\_t} \tag{8}$$

The actual form of (7) needs to take into account the nature of our industry and the historical empirical relationship between oil price and our variable cost. We note that the form of (7) does not matter in illustrating our framework. What is important is to establish a tractable analytical function for variable cost *VC* thereby we can use it to facilitate our Monte Carlo simulation later on. Here for implementation purpose, we choose $a_V = 1$ and $b_V = 0.8$.

### 3.4. Battery Price $p_t$

The cost of an automotive lithium-ion battery pack was between $1,000 and $1,200 per kWh in 2009. The consumer batteries were costing lower ($250 to $400 per kWh) because of the difference in production complexity and technology according to, e.g., Dinger et al (2009). They also pointed out that the price tag will likely decline to between $250 and $500 per kWh at scaled production. Since consumer batteries are simpler and must have much less demanding requirements concerning safety and life span for instance, it is reasonable to consider $250 to $400 to be long-term equilibrium price range for a pack of automotive lithium-ion batteries. Indeed, Dinger et al (2009) claimed that $250 has persisted as the cost goal for a pack. Nevertheless, they also acknowledged the challenges to achieving this goal by 2020. At the time of this writing (February 2015), such a pack costs $500 per kWh.

The above numbers give us a general picture of how the battery price has evolved in the past and will evolve in the future. It is very much like most other products as they undergo the process of innovation, maturity, competitive advantage, scaled production, and very thin profit margin. This is because the current technology is facing competition continuously and the battery price will go down as time goes on when more competitors are entering the arena. In theory, the net flow of newcomers stabilizes when the NPV is about zero for a marginal newcomer, and later on the technology will phase out when the NPV becomes negative for the existing players using the outdated technology for production. Therefore, we assume that the current technology will reach complete maturity and maximum efficiency in *H* years (say, 5 years), by which we mean that the price will be barely covering variable cost, i.e., $p_{H\ years} = VC$. This is because fixed cost is irrelevant once the facility has been built. And then, when new technology starts to compete, the price will be significantly driven down which makes the existing new technology completely outdated. We assume this scenario will take place around 1.5*H* years, i.e., 7.5 years in the current discussion. Obviously, our firm will keep using the current technology as long as its net profit is positive or when our preset terminal time is reached. When "stop" is triggered, all cash flows become zero. For simplicity, we assume away any scrapping value or shut-down cost.

Now, we assume that the battery price is given by the following piecewise function,

$$p_t = p_{b\_0} - \left(p_{b\_0} - VC_t\right) \times e^{\frac{t-H}{t}}, \quad \forall t \le H \quad \text{profitable} \tag{9a}$$

$$p_t = VC_t, \quad \forall t \in [H, 2H] \quad \text{may still hang on} \tag{9b}$$

$$p_t = 0.5VC_t, \quad \forall t \in [2H, \infty] \quad \text{absolutely outdated} \tag{9c}$$

Figure 3 shows the behavior of this piecewise function by choosing initial battery price $p_0 = \$1{,}000$ per kWh, $VC_t$ is fixed or stochastic and the expected value is, say, \$400 per kWh at horizon $H$.

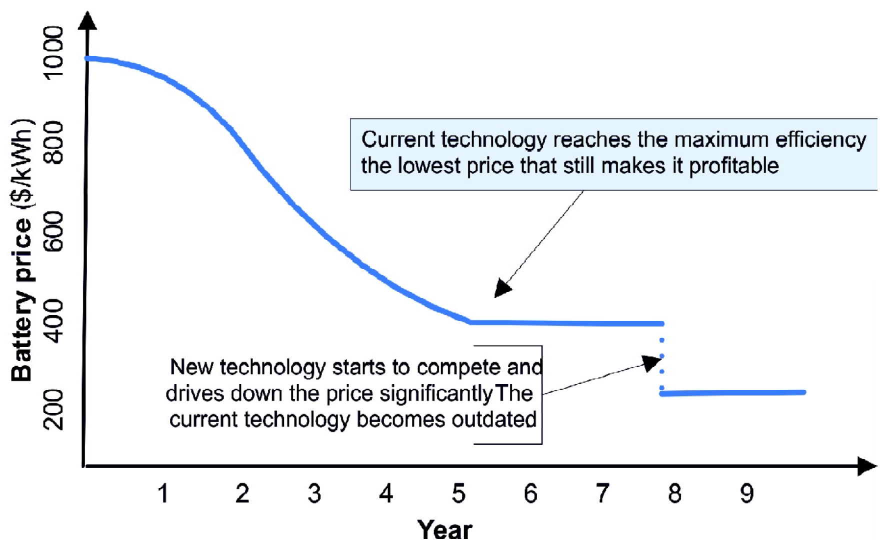


**Figure 3: Price of a pack of lithium-ion automotive batteries is a declining function of time. We assume that the price will stabilize slightly above the long-term variable cists**

In reality, battery price may likely exhibit some fluctuations. It would be more realistic if we add a Brownian motion-like component to account for any anticipated noise in price specified in (9). However, as we show later on, what we are interested in ultimately is the expected NPVs and prices, therefore, for simplicity we do not include the noise component in battery price in the current implementation. Nevertheless, it is quite straightforward to consider such noise components in the later numerical computation.

### 3.5. Discount rate $\Re_t$

We use the CAPM to determine the appropriate discount rate as in (10) in its general form,

$$r = r_f + \beta\left(r_M - r_f\right) \tag{10}$$

where $\left(r_M - r_f\right)$ can be approximated by the average market risk premium up to the current point, and riskfree rate $r_f$ can be treated likewise. The easy form of (10) belies the considerable difficulties behind the surface. Appropriately determining the discount rate is one of the major challenges in this framework. This is because the revenues, costs, profits and free cash flows all change from period to period depending a variety of factors and some of them are stochastic.

The asset value (or value from operations) is the present value of the free cash flows from running the business of making and selling lithium-ion batteries. The appropriate discount rate must reflect business risk and financial risk. Again, for simplicity, we neglect debt by treating our firm as all-equity financed. To consider financial leverage and its impact on the discount rate, one may use the MM capital structure argument as an approximation. To further consider tax effect and risk cash flows, please see, e.g., Qi, Liu and Johnson (2012).

After assuming away financial leverage, the total risk only comes from business risk which has two contributors – the fluctuations in revenue, and operating leverage. The former is largely a macroeconomic factor, while the latter is more of a firm management-specific microeconomic factor. It can be shown, using the most widely used quadratic utility function in financial economics, that the appropriate beta may be given as follows (see Appendix, section 3 for proof),

$$\beta_{\text{Rev}\_t} = \frac{\dfrac{M_t + \mu_{\text{Asset}\_t}}{v_t} - 1 - r_f}{\overline{r_M - r_f}} = \frac{\dfrac{M_t + \mu_{\text{Asset}\_t}}{v_t} - 1 - r_f}{r_M - r_f} \tag{11}$$

We also prove that for 1 time period, i.e., [$t$, $t$ + 1], the relationship between $\beta_{\text{Assets}\_t}$ and $\beta_{\text{Rev}\_t}$ is determined via equation (12),

$$\beta_{\text{Assets}\_t} \approx \beta_{\text{Rev}\_t}\left\{1 + l_t \times \left[\frac{p_{t+1}Q_{t+1}}{\dfrac{FC_{t+1}}{1+r_f}R_{\text{Rev}\_t} + VC_{t+1}Q_{t+1}} - 1\right]^{-1}\right\} \tag{12}$$

Where the corresponding operating leverage $l_t$ is [See Appendix A3.7]

$$l_t = \text{Operating Leverage} \approx \frac{PV(FC_{t+1})}{PV(FC_{t+1}) + PV(VC_{t+1} \times Q_{t+1})}$$
$$= (1 + \frac{VC_{t+1}Q_{t+1}}{\left(1 + \beta_{\text{Rev}} \frac{r_M - r_f}{1 + r_f}\right) \times FC_{t+1}}\}^{-1} \tag{13}$$

And

$$R_{\text{Rev\_}t} = 1 + r_f + \beta_{\text{Rev\_}t}\left(r_M - r_f\right) \tag{14}$$

Therefore, the discount rate for asset cash flow from $t + 1$ to $t$ is

$$r_t \equiv r_{\text{Asset\_}t} = r_f + \beta_{\text{Asset\_}t}\left(r_M - r_f\right) \tag{15}$$

Discount rate $r_t$ will be used to discount asset cash flow from $t + 1$ to $t$. We note that $r_t$ is uniquely determined by oil price $p_{oil\_t}$ via variable cost $VC_t$ and stochastic demand $Q_t$.

Therefore, the appropriate spot rate ($\Re_t$) for asset cash flow $CF_t = \left(p_t - VC_t\right) \times Q_t - FC_t$ occurring at time $t$ is finally given by

$$\Re_t \equiv \left[\prod_{i=1}^{t}\left(1 + r_{\text{Asset\_}i}\right)\right]^{\frac{1}{t}} - 1 \tag{16}$$

Finally, Figure 4 highlights the increasing effect of operating leverage on both business risk and the corresponding risk-adjusted discount rate ($r_t$) based on the CAPM. Since demand ($Q_t$) and variable cost $VC_t$ are stochastic, the resulting operating leverage is a function of time, therefore, the appropriate discount rate ($r_t$) is expected to evolve in time as well. Our model captures these effects and their interactions in (16), which is sufficient to allow us to finally solve equation (2) and ultimately our goal of equation (1).

## IV. CONCLUSIONS

Corporate actions to raise revenue do not necessarily lead to higher profits or higher firm value. This is because expansion of production will increase fixed costs and therefore operating leverage is likely to go up. In this study, we show

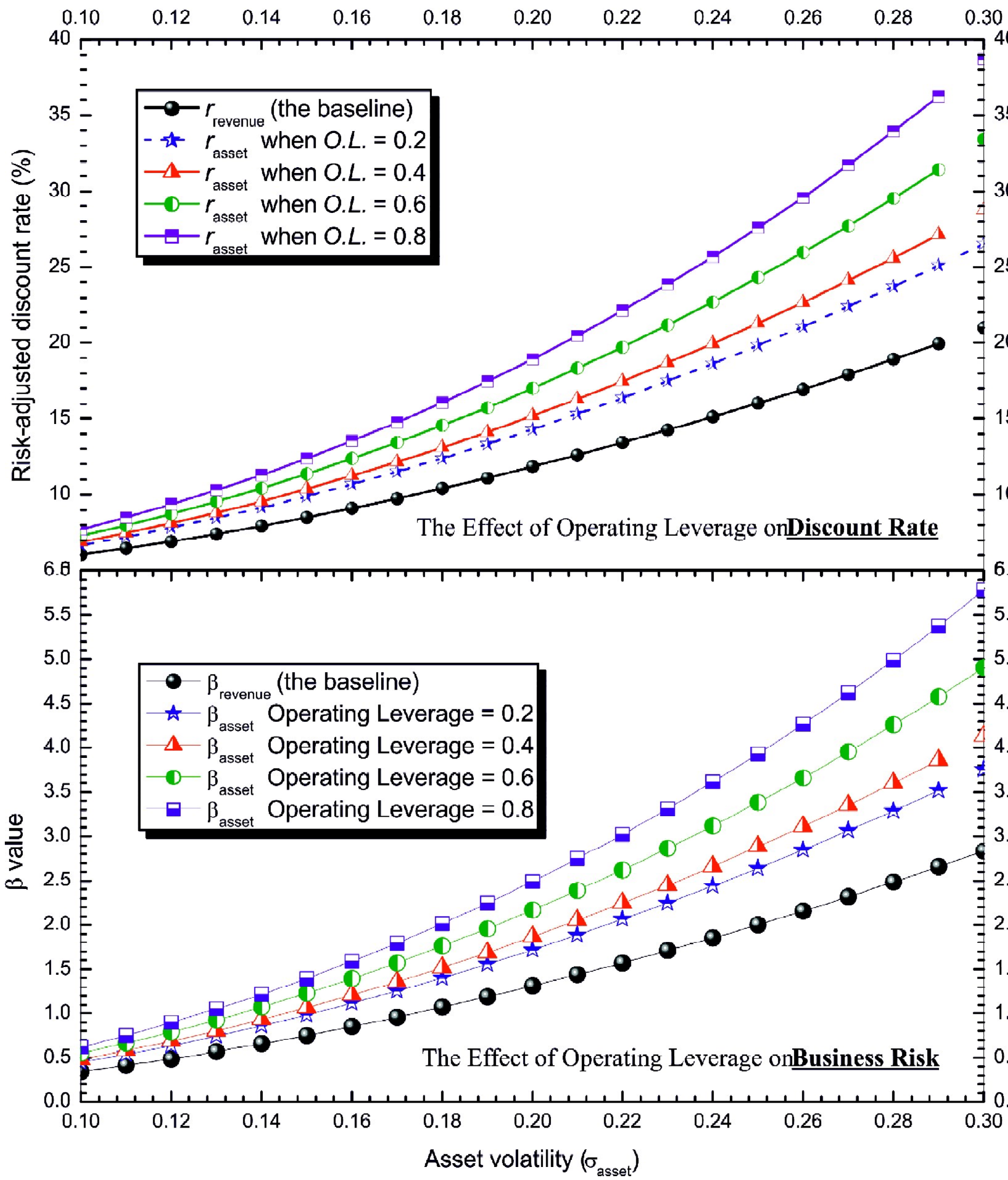


**Figure 4: The effect of operating leverage on business risk and the corresponding risk-adjusted discount rate. For illustration purpose, we choose $r_M$ = 10%, $r_f$ = 4%, demand $Q$ = 100 and capacity $Q_{capacity}$ = 120.**

that higher operating leverage will increase firms' business risk and the risk-adjusted discount rate. These effects will collectively exert a downward pressure on firm's value. But on the other hand, increased production scale means higher revenues which are normally necessary in boosting firm value. Therefore, increasing production capacity can have the aforementioned two competing forces on firm value. Which one of them dominates comes down to the optimization of capacity. When demand is uncertain, this becomes considerably challenging.

In this study, we present a structural model that solves this capacity optimization problem when firms facing uncertain demand caused by a stochastic driving force. We use lithium-ion batteries used in almost every electric and hybrid cars as a vehicle to illustrate the development and implementation of the framework we propose.

The stochastic driving force is oil price which is assumed to follow the geometric Brownian motion. Lithium-ion battery price is assumed to drop in time with the describing function that can be calibrated against the observed data. Demand, cost, revenue, operating leverage, business risk and the appropriate discount rate, etc. are all affected by oil price fluctuations and interconnected endogenously within our model. These factors jointly help decide the optimal production capacity for our lithium-ion batteries.

Our contribution is to make available a tool that considers various relevant factors in a theoretically consistent framework and quantitatively set such a threshold known as the optimal capacity. Our model is easy to implement for firms in the real world to help answer a critical question that firms tend to face from time to time – when to happily accept an order for production, and when to politely refuse a big order. Our current model does not consider the impact of inventory which certainly has rich features to explore in our context. We leave the joint optimization of inventory and capacity as a natural continuation for future research.

# APPENDIX

## Detailed Derivations of All the Relationships Presented in This Study

### 1. Operating leverage and business risk

*1.1. Business risk is characterized by* $\frac{\partial(EBIT)}{\partial(Sales)} \approx \text{Operating Leverage} = \frac{PV(\text{fixed costs})}{PV(\text{total costs})}$

Proof.

Business risk reflects the sensitivity of EBIT to changes in sales. Therefore, it is normally defined as

$$\frac{\Delta(EBIT)}{\Delta(Sales)} \tag{A1.1}$$

Suppose $P$ is the price, Q is the quantity sold, V and F are variable and fixed costs, respectively. Since EBIT = P*Q – V*Q – F, we have

$$\frac{\partial(EBIT)}{\partial(Sales)} = \frac{\Delta(P \times Q - V \times Q - F)}{\Delta(P \times Q)} = 1 - \frac{\Delta(V \times Q)}{\Delta(P \times Q)} \tag{A1.2}$$

Assume variable costs per product ($V$) and the price per product ($P$) do not change, then

$$\frac{\partial(EBIT)}{\partial(Sales)} = 1 - \frac{\Delta(V \times Q)}{\Delta(P \times Q)} = 1 - \frac{V \times \Delta Q}{P \times \Delta Q} = 1 - \frac{V}{P} \tag{A1.3}$$

Since $P = f + V + ebit$ where $ebit$ is the (gross business) profit per product, and $f = F/Q$,

$$\frac{\partial(EBIT)}{\partial(Sales)} = 1 - \frac{V}{f + V + ebit} = \frac{f + ebit}{f + V + ebit} \tag{A1.4}$$

Assume $P = f + V + ebit \approx f + V$ if $ebit$ is small, then

$$\frac{\partial(EBIT)}{\partial(Sales)} = \frac{f + ebit}{f + V + ebit} \approx \frac{f}{f + V} \tag{A1.5}$$

Notice, in the above derivations, the values are all in the present value sense, therefore, we have proved the following claim

$$\frac{\partial(EBIT)}{\partial(Sales)} \approx \frac{f}{f + V} = \frac{PV\,(\text{fixed costs})}{PV\,(\text{total costs})} = \text{Operating Leverage} \tag{A1.6}$$

## 1.2. Operating leverage and the betas

We note that

PV(Revenue) = PV(Fixed cost) + PV(Variable cost) + PV(Asset or EBIT)

These values can serve as the weights for calculating revenue beta on the left-hand-side,

$$\beta_{\text{revenue}} = \beta_{\text{fixed cost}} \frac{\text{PV(fixed cost)}}{\text{PV(revenue)}} + \beta_{\text{variable cost}} \frac{\text{PV(variable cost)}}{\text{PV(revenue)}} + \beta_{\text{asset}} \frac{\text{PV(asset)}}{\text{PV(revenue)}} \tag{A1.7}$$

It is reasonable to assume their corresponding betas have the following values,

$$\beta_{\text{revenue}} = \beta_{\text{ariable costs}} \text{ and } \beta_{\text{fixed cost}} = 0 \tag{A1.8}$$

Now we can simplify revenue beta as follows,

$$\beta_{\text{asset}} = \beta_{\text{revenue}} \times \frac{PV(\text{rev}) - PV(\text{v.c.})}{PV(\text{asset})} = \beta_{\text{revenue}} \times \frac{PV(\text{f.c.}) + PV(\text{v.c.}) + PV(\text{asset}) - PV(\text{vc})}{PV(\text{asset})}$$

$$= \beta_{\text{revenue}} \times \left[1 + \frac{PV(\text{f.c.})}{PV(\text{asset})}\right] = \beta_{\text{revenue}} \times \left[1 + \frac{PV(\text{f.c.})}{PV(\text{rev}) - PV(\text{t.c.})}\right] = \beta_{\text{revenue}} \times \left[1 + \frac{\frac{PV(\text{f.c.})}{PV(\text{t.c.})}}{\frac{PV(\text{rev})}{PV(\text{t.c.})} - 1}\right]$$

$$= \beta_{\text{revenue}} \times \left[1 + \frac{\text{Operating Leverage}}{\frac{PV(\text{rev})}{PV(\text{t.c.})} - 1}\right] \tag{A1.8}$$

Where $PV(\text{f.c.})$, $PV(\text{v.c.})$, $PV(\text{t.c.})$ and $PV(rev)$ are the present value of fixed cost, variable cost, total cost, and revenue, respectively; and

$$\text{Operating Leverage} = \frac{PV(\text{f.c.})}{PV(\text{t.c.})} \tag{A1.9}$$

Clearly, asset risk, or business risk, characterized by $\beta_{asset}$ is an increasing function of operating leverage.

## 2. QUADRATIC UTILITY FUNCTION, RISK AVERSION, PV AND BETA

### 2.1. The CAMP and Quadratic Utility Function

The CAPM is built on the assumption of quadratic utility function, or mean-variance utility function. In other words, investors' risk aversion and their valuation of risky assets are adequately described by a utility function in quadratic form as follows,

$$U(M) = M - kM^2 \tag{A2.1}$$

Where $M$ is wealth, which is the uncertain cash flow in our case. The utility function's derivatives are

$$U'(M) = 1 - 2kM \quad \text{and} \quad U''(M) = -2k \tag{A2.2}$$

If the investor is risk averse, then the second derivative is negative, i.e., $k > 0$. If the investor has increasing marginal utility, i.e., nonsatiation, then

$$U'(M) = 1 - 2kM > 0 \tag{A2.3}$$

Denoting variance of $M$ as

$$\sigma_M^2 = E\left[M^2\right] - \left(E[M]\right)^2 \tag{A2.4}$$

Then,

$$E\left[U(M)\right] = E[M] - k\left\{\sigma_M^2 + \left(E[M]\right)^2\right\} \tag{A2.5}$$

The above equation shows that the expected utility is strictly defined in terms of means and variances. This is why in financial economics, quadratic-form utility function is the most frequently used to describe investor's behavior.

## 2.2. Determine the Revenue Cash Flow Process and its Conditional Statistics

The revenue cash flow $CF_t$ over time period $\left[t-1, t\right]$ is

$$CF_t = p_t Q_t \tag{A2.6}$$

For $1 \le t \le H$, we replace battery price $p_t$ with equation (9a) to arrive at

$$CF_t = \left[p_{b\_0} - p_{b\_0} e^{\frac{t-H}{t}} + VC_t e^{\frac{t-H}{t}}\right] Q_t \tag{A2.7}$$

Differentiating (A2.7) with ItM's lemma to have

$$\begin{aligned} dCF_t = &\left[p_{b\_0} - p_{b\_0} e^{\frac{t-H}{t}} + VC_t e^{\frac{t-H}{t}}\right] dQ_t + \left[- p_{b\_0} Q_t + VC_t Q_t\right] e^{\frac{t-H}{t}} \frac{H}{t^2} dt \\ &+ Q_t e^{\frac{t-H}{t}} dVC_t \end{aligned} \tag{A2.8}$$

$$\begin{aligned} = &\left[p_{b\_0} - p_{b\_0} e^{\frac{t-H}{t}} + VC_t e^{\frac{t-H}{t}}\right] \times \left[\left(b_e + \frac{c_e}{\sqrt{p_{oil\_t}}}\right) dp_{oil\_t} - \frac{\left(p_{oil\_t}\right)^{-3/2}}{4} \left(dp_{oil\_t}\right)^2\right] \\ &+ \left[- p_{b\_0} Q_t + VC_t Q_t\right] e^{\frac{t-H}{t}} \frac{H}{t^2} dt \\ &+ Q_t e^{\frac{t-H}{t}} \left(\frac{b_V}{p_{oil\_t}} dp_{oil\_t} + \frac{b_V}{2\left(p_{oil\_t}\right)^2} \left(dp_{oil\_t}\right)^2\right) \end{aligned}$$

Rearrange the above equation to have

$$dCF_t = \left[\left(p_{b\_0} - p_{b\_0}e^{\frac{t-H}{t}} + VC_t e^{\frac{t-H}{t}}\right)\times\left(b_e + \frac{c_e}{\sqrt{p_{oil\_t}}}\right) + Q_t e^{\frac{t-H}{t}}\frac{b_V}{p_{oil\_t}}\right]dp_{oil\_t}$$
$$+\left[Q_t e^{\frac{t-H}{t}}\frac{b_V}{2\left(p_{oil\_t}\right)^2} - \left(p_{b\_0} - p_{b\_0}e^{\frac{t-H}{t}} + VC_t e^{\frac{t-H}{t}}\right)\times\frac{\left(p_{oil\_t}\right)^{-3/2}}{4}\right]\left(dp_{oil\_t}\right)^2$$
$$+\left[-p_{b\_0}Q_t + VC_t Q_t\right]e^{\frac{t-H}{t}}\frac{H}{t^2}dt \tag{A2.9}$$

Further simplifying the notation with

$$\begin{cases}\delta_t = \left(p_{b\_0} - p_{b\_0}e^{\frac{t-H}{t}} + VC_t e^{\frac{t-H}{t}}\right)\times\left(b_e + \frac{c_e}{\sqrt{p_{oil\_t}}}\right) + Q_t e^{\frac{t-H}{t}}\frac{b_V}{p_{oil\_t}} \\ \phi_t = Q_t e^{\frac{t-H}{t}}\frac{b_V}{2\left(p_{oil\_t}\right)^2} - \left(p_{b\_0} - p_{b\_0}e^{\frac{t-H}{t}} + VC_t e^{\frac{t-H}{t}}\right)\times\frac{\left(p_{oil\_t}\right)^{-3/2}}{4} \\ \lambda_t = \left[-p_{b\_0}Q_t + VC_t Q_t\right]e^{\frac{t-H}{t}}\frac{H}{t^2}\end{cases} \tag{A2.10}$$

Therefore

$$dCF_t = \delta_t dp_{oil\_t} + \phi_t\left(dp_{oil\_t}\right)^2 + \lambda_t dt \tag{A2.11}$$

Inserting $p_{oil\_t}$ process, $dp_{oil\_t} = \mu_{oil}\, p_{oil\_t}dt + \sigma_{oil}\, p_{oil\_t}dW_t$, then

$$\begin{aligned}dCF_t &= \delta_t\mu_{oil}\, p_{oil\_t}dt + \delta_t\sigma_{oil}\, p_{oil\_t}dW_t + \phi_t\left(\sigma_{oil}\, p_{oil\_t}\right)^2 dW^2 + \lambda_t dt \\ &= \left(\delta_t\mu_{oil}\, p_{oil\_t} + \phi_t\left(\sigma_{oil}\, p_{oil\_t}\right)^2 + \lambda_t\right)dt + \delta_t\sigma_{oil}\, p_{oil\_t}dW\end{aligned} \tag{A2.12}$$

This specifies the asset cash flow process. We simplify it to

$$dM_t = dCF_t = \mu_{\text{Asset}\_t}dt + \sigma_{\text{Asset}\_t}dW_t \tag{A2.13}$$

With

$$\begin{cases} \mu_{\text{Asset}\_t} = \delta_t \mu_{oil} p_{oil\_t} + \phi_t \left(\sigma_{oil} p_{oil\_t}\right)^2 + \lambda_t^{\ 2} \\ \sigma_{\text{Asset}\_t} = \delta_t \sigma_{oil} p_{oil\_t} \end{cases} \tag{A2.14}$$

In discrete form (when the Monte Carlo simulation is carried out), we replace $dt$ with $\Delta t$. Now let

$$M_{t+\Delta t} = M_t + \Delta M_t = M_t + \mu_{\text{Asset}\_t} \Delta t + \sigma_{\text{Asset}\_t} \Delta W_t \tag{A2.15}$$

Therefore, it is easy to find the conditional statistics based on information available at $t$

$$\begin{cases} E_t\left[\left(M_{t+\Delta t}\right)^2\right] = M_t^2 + \left(2M_t \mu_{\text{Asset}\_t} + \sigma_{\text{Asset}\_t}^2\right)\Delta t \\ E_t\left[M_{t+\Delta t}\right] = M_t + \mu_{\text{Asset}\_t} \Delta t \end{cases} \tag{A2.16}$$

According to (A2.4) and (A2.5), and simplifying the notation by defining $\sigma_M^2 \equiv \sigma_M^2 \mid_{t \to t+\Delta t}$ as the expected volatility over the time period $[t, t+\Delta t]$, we have

$$\begin{aligned} \sigma_M^2 &\equiv \sigma_M^2 \mid_{t \to t+\Delta t} = E_t\left[M_{t+\Delta t}^2\right] - \left(E_t\left[M_{t+\Delta t}\right]\right)^2 \\ &= M_t^2 + \left(2M_t \mu_{\text{Asset}\_t} + \sigma_{\text{Asset}\_t}^2\right)\Delta t - M_t^2 - 2M_t \mu_{\text{Asset}\_t} \Delta t - \left(\mu_{\text{Asset}\_t} \Delta t\right)^2 \\ &= \sigma_{\text{Asset}\_t}^2 \Delta t \end{aligned} \tag{A2.17}$$

And

$$\begin{aligned} E_t\left[U\left(M_{t+\Delta t}\right)\right] &= E_t\left[M_{t+\Delta t}\right] - k\left\{\sigma_M^2 + \left(E_t\left[M_{t+\Delta t}\right]\right)^2\right\} \\ &= M_t + \mu_{\text{Asset}\_t} \Delta t - k\left\{\sigma_{\text{Asset}\_t}^2 \Delta t + M_t^2 + 2M_t \mu_{\text{Asset}\_t} \Delta t + \left(\mu_{\text{Asset}\_t} \Delta t\right)^2\right\} \\ &= M_t - kM_t^2 + \left(\mu_{\text{Asset}\_t} - k\sigma_{\text{Asset}\_t}^2 - 2kM_t \mu_{\text{Asset}\_t}\right)\Delta t \end{aligned}$$

(A2.18)

## 3. DISCOUNT RATE FOR ASSET CASH FLOWS $CF_t$

Now to work in the discrete form, we choose 1 time period in the discrete form over the time period of $\Delta t = [t, t+1]$. Suppose the present value of risk asset cash flow $M_{t+1}$ is $v_t$ whose certainty-equivalent future value is $v_t\left(1 + r_f\right)$ at $t+1$, then according to (A2.5), at $t + 1$ the utilities must satisfy the following equality, by definition,

$$E_t\left[U\left(v_{t+1}\right)\right] = v_t\left(1 + r_f\right) - kv_t^2\left(1 + r_f\right)^2 \equiv E_t\left[U(M_{t+1})\right] \tag{A3.1}$$

Solving (A3.1) for $v_t$ to have

$$v_t = \frac{1 + \sqrt{1 - 4kE_t\left[U(M_{t+1})\right]}}{2k\left(1 + r_f\right)} \tag{A3.2}$$

Alternatively, based on (A2.16), we have $v_t = \dfrac{E_t\left[M_{t+1}\right]}{1 + r_t} = \dfrac{M_t + \mu_{\text{Asset}\_t}\Delta t}{1 + r_t} = \dfrac{M_t + \mu_{\text{Asset}\_t}}{1 + r_t}$, where $r_t = r_f + \beta_{\text{Asset}\_t}\left(\overline{r_M - r_f}\right)$ and $\left(\overline{r_M - r_f}\right)$ is the expected risk premium. For practical purposes, $\left(\overline{r_M - r_f}\right)$ is equal to the current observed market risk premium. This is also because they are the unbiased estimators for future rates given the common assumption of Brownian motion. Thus,

$$\beta_{\text{Asset}\_t} = \frac{\dfrac{M_t + \mu_{\text{Asset}\_t}}{v_t} - 1 - r_f}{\overline{r_M - r_f}} = \frac{\dfrac{M_t + \mu_{\text{Asset}\_t}}{v_t} - 1 - r_f}{r_M - r_f} \tag{A3.3}$$

And the discount rate for asset cash flow from $t + 1$ to $t$ is

$$r_t \equiv r_{\text{Asset}\_t} = r_f + \beta_{\text{Asset}\_t}\left(r_M - r_f\right) \tag{A3.9}$$

Discount rate $r_t$ will be used to discount asset cash flow from $t + 1$ to $t$, and the spot rate ($\Re_t$) for asset cash flow $CF_t = \left(p_t - VC_t\right) \times Q_t - FC_t$

$$\Re_t \equiv \left[\prod_{i=1}^{t}\left(1 + r_{\text{Asset}\_i}\right)\right]^{\frac{1}{t}} - 1 \tag{A3.10}$$

That is

$$PV\left(CF_t\right) = \frac{\left(p_t - VC_t\right) \times Q_t - FC_t}{\left(1 + \Re_t\right)^t} \tag{A3.11}$$

The total net value of the project is therefore,

$$V_T = \sum_{t=1}^{T} PV(CF_t) \tag{A3.12}$$

If we further require that the battery project stop by the horizon $H$ because for $t > H$ the annual profit becomes zero without considering fixed cost (since the facility has been installed already) or negative if fixed cost is considered. Therefore, it makes better sense to scrap the project and switch to the new technology. If this condition is satisfied, then our firm's ultimate optimization process becomes

$$Q*_{Capacity} = \underset{Q_{Capacity}}{Arg \quad Max}[V_T] = \underset{Q_{Capacity}}{Arg \quad Max}\left[\sum_{t=1}^{T} PV(CF_t)\right], \quad s.t. \quad T \leq H \tag{A3.13}$$

Where $T$ is the life of the lithium project.

## 4. THE FINAL RESULT IN MONTE CARLO SIMULATION

When we implement the model via a Monte Carlo simulation, we generate $N$ sets of time series of oil prices $\{p_{oil\_0}, p_{oil\_1}, \ldots, p_{oil\_t}, \ldots, p_{oil\_T}\}$, and the optimal capacity $Q*_{Capacity}$ in (A3.13) can be used to generate the expected value in the statistical sense, and define the final optimized production capacity as $Q* \equiv E[Q*_{Capacity}]$ which is given below,

$$Q* \equiv E[Q*_{Capacity}] = \underset{Q_{Capacity}}{Arg \quad Max}[E[V_T]] \approx \frac{\sum_{i=1}^{N} Q*_{Capacity\_i}}{N} \tag{A4.1}$$

The final decision rule becomes – if $Q_t < Q*$, our firm should gladly accept the order, and when $Q_t \geq Q*$, it should politely refuse to take the order.

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